Preparation for EMC 2024

Fourth Training Test for Junior Category

9th December 2024

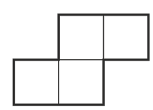
Problem 1. In acute-angled $\triangle ABC$ with $AB < BC$, points M, N are the midpoints of sides AB, AC respectively. Let D and E be two points on the segment BN such that $CD = 2ME$ and $BE < ED$. Prove that $\angle NEM = \angle CDN$.

Problem 2. Find all positive integers n for which

$$\frac{2^{n!}-1}{2^n-1}$$

is a perfect square.

Problem 3. Let $n \geq 3$ be an integer. In a square of size $n \times n$ we place the shapes on the picture on the right, such that the unit squares coincide. On each unit square it is allowed to overlap at most two figures, rotations and flips are allowed, and no figure can exit the borders of the square. For each n find the least number of unit squares that must be left uncovered (a square is covered if at least one figure covers it).



Problem 4. **Problem 4.** Let $a \geq b \geq c \geq d$ be positive real numbers. Prove that

$$\frac{b^3}{a} + \frac{c^3}{b} + \frac{d^3}{c} + \frac{a^3}{d} + 3\,(ab + bc + cd + da) \geq 4\big(a^2 + b^2 + c^2 + d^2\big).$$

When does the equality hold?

Allotted time: 4 hours.