Preparation for EMC 2024

Fourth Training Test for Senior Category

9th December 2024

Problem 1. On an infinite chessboard consisting of unit squares (x, y) with $x, y \geq 0$ two players play the following game: initially a king is positioned somewhere on the board, but not on $(0, 0)$, and they alternatively move it either down or left or down-left. The player Who loses is the one who moves the king into the $(0, 0)$ square. Find the initial position of the king for which the first player wins.

Problem 2. Let n be a positive integer. What is the largest number of elements that one can choose from the set $A = \{1, 2, \ldots, 2n\}$ such that the sum of any two chosen numbers is composite?

Problem 3. Prove that for every positive integer n there exists an n-digit number divisible by 5^n, all of whose digits are odd.

Problem 4. Determine all functions $f : \{1, 2, 3, \ldots\} \to \{1, 2, 3, \ldots\}$ which satisfy the property that for all positive integers a and b, there exists a nondegenerate triangle with sides of lengths

$$a, f(b) \text{ and } f(b + f(a) - 1).$$

(A triangle is non-degenerate if its vertices are not collinear.)

Allotted time: 4 hours.