

Preparation for EMC 2024

Third Training Test for Junior Category

8th December 2024

Problem 1. Let a , b , and c be real numbers greater or equal to 1. Prove that

$$\frac{ab}{c-1} + \frac{bc}{a-1} + \frac{ca}{b-1} \geq 12.$$

When does equality hold?

Problem 2. In a country there are only four type of coins valued at 74, 87, 111, and 124 dollars respectively. In how many different ways can one pay exactly 2023 dollars?

Two ways are different if there are different numbers of coins of at least one type between them.

Problem 3. Let $\triangle ABC$ be scalene with circumcircle ω and incentre I . The tangent of ω at C meets AB at D and the bisector of $\angle BDC$ meets BI and AI at P and Q respectively. If M is the midpoint of the segment PQ , prove that the line IM passes through the midpoint of the arc AB of ω that contains C .

Problem 4. Determine all triplets of integers (p, x, y) , such that p is a prime, $p - 1 = x^2$, and $2p^2 - 1 = y^2$.

Allotted time: 4 hours.