# Preparation for EMC 2024

Second Training Test for Junior Category

1st December 2024

**Problem 1.** Two players play a game where they take turns to make a move and which always finishes with the victory of one of the players. The game is also designed so that it ends in at most $n$ steps, for some fixed positive integer n. Prove that either the first or the second player has a Winning strategy.

**Problem 2.** Let $ABC$ be an acute-angled triangle with circumcircle $\Gamma$ and orthocenter $H$. Let $K$ be a point of $\Gamma$ on the other side of $BC$ from $A$. Let $L$ be the reflection of $K$ in the line $AB$, and let $M$ be the reflection of $K$ in the line $BC$. Let $E$ be the second point of intersection of $\Gamma$ with the circumcircle of triangle $BLM$. Show that the lines $KH$, $EM$ and $BC$ are concurrent.

**Problem 3.** Prove that every positive integer $n$ is a sum of one or more numbers of the form $2^r 3^s$, where $r$ and $s$ are non-negative integers and no summand divides another (for example, $23 = 9 + 8 + 6$).

**Problem 4.** If $0 < a \leq b \leq c \leq d$, prove that

$$a^b b^c c^d d^a \geq b^a c^b d^c a^d \, .$$

*Allotted time: 4 hours.*