Preparation for EMC 2024

First Training Test for Junior Category

24th November 2024

Problem 1. Determine whether there is a natural number n for which $8^n + 47$ is prime.

Problem 2. Let $a, b, c, d > 0$ such that $a + b + c + d = 1$. Prove the inequality:

$$\frac{1}{4a + 3b + c} + \frac{1}{3a + b + 4d} + \frac{1}{a + 4c + 3d} + \frac{1}{4b + 3c + d} \geq 2.$$

Problem 3. Let $ABCD$ be a convex quadrilateral and let M be the midpoint of the side CD. Lines BM and AM are perpendicular. Prove that lines BC and AD are parallel if and only if, $AB = BC + AD$.

Problem 4. Consider a 2×2011 table. Two players in turn place dominoes on it, the first one places only horizontal dominoes and the second one places only vertical dominoes. The dominoes may not overlap. The player who has no legal move loses the game. Which player has a winning strategy?

Allotted time: 4 hours.